Rummler Channel Model used in RI 241

Application Note

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Contents

1 General

1.1 Terminology / Abbreviations / Symbols

ADC	Analog-to-Digital Converter
Amp	Amplifier
DAC	Digital-to-Analog Converter
FPGA	Field Programmable Gate Array
GUI	Graphical User Interface
NA	Not Applicable
TBD	To Be Decided or To Be Defined

2 Introduction

Rummler model is a multipath fading model developed for terrestrial communication links between fixed antenna towers. The channel is a line-of-sight radio channel. Rummler model is widely used for terrestrial microwave links.

An example of the Rummler model frequency response can be seen in fig. 2.1. The parameters of the Rummler Model, that determine notch frequency, group delay, etc., are random variables with distributions that have been empirically determined.

But the parameters of Rummler's fading channel model in RI 241 are controlled by the user, and are not random variables. The parameters are usually swept/stepped in a deterministic way in order to e.g. carry out Signature Tests. Such a Signature Test is specified in the terrestrial microwave link standard ETSI EN 302 217-2-1 V2.0.1 at the section *Distortion sensitivity*.

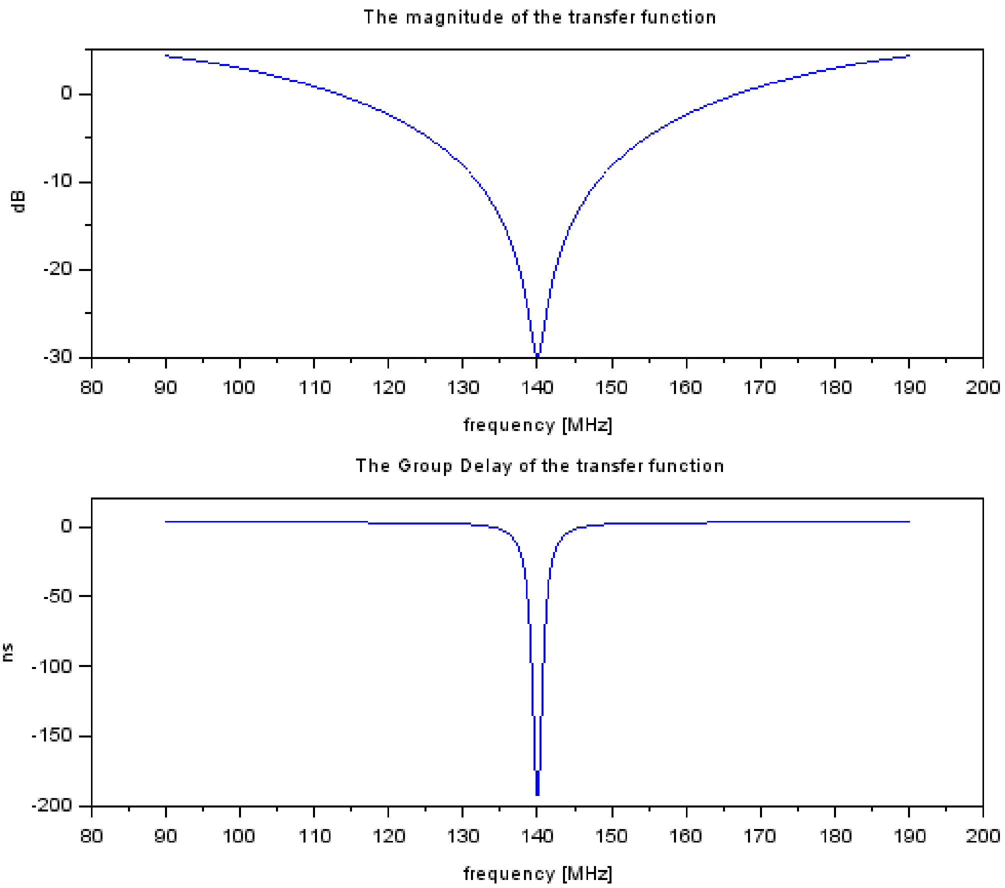

Fig. 2.1 An example of the frequency response of Rummler's fading channel model in RI 241. The Notch Depth is 30 dB at 140 MHz and the Group Delay is -193 ns at 140 MHz.

3 Rummler Channel Model

The fading model to be implemented is Rummler channel model. The Rummler channel model is effectively a two-path model. The transfer function *H*(*f*) of Rummler channel model can be written as follows

$$H(f) = \alpha - \beta e^{-j2\pi(f-f_0)\tau} \tag{3.1}$$

or the impulse response *h*(*t*) as

$$h(t) = \alpha\delta(t) - \beta e^{j2\pi f_0\tau}\delta(t-\tau) \tag{3.2}$$

Where

f = the frequency.

f_0 = the notch frequency.

α = the parameter associated with the attenuation of the direct signal path.

β = the parameter associated with the attenuation of the reflected signal path.

τ = the time delay associated with the interpath time delay difference of the two paths (direct and reflected) in the model.

The transfer function is *minimum-phase* when $\alpha > \beta$ and *nonminimum-phase* when $\alpha \le \beta$. Rummler Channel Model is implemented in an FPGA, see fig. 3.1 below, and the gain/attenuation from IF IN to IF OUT is adjusted with fast analog attenuators. Therefore it's the magnitude of α relative β that is of interest in the implementation in the FPGA. So equ. (3.1) are rewritten as follows

$$H(f) = \alpha\left[1 - \frac{\beta}{\alpha}e^{-j2\pi(f-f_0)\tau}\right],\ \alpha > \beta \tag{3.3}$$

$$H(f) = \beta\left[\frac{\alpha}{\beta} - e^{-j2\pi(f-f_0)\tau}\right],\ \alpha \le \beta \tag{3.4}$$

In equ. (3.3) it's the attenuation of the direct path that is chosen to be the reference, so α is set to $\alpha = 1$, and in equ. (3.4) it's the attenuation of the reflected path that is chosen to be the reference, so β is set to $\beta = 1$. That gives us

$$H(f) = 1 - \beta e^{-j2\pi(f-f_0)\tau},\ \beta < 1,\ \textit{minimum-phase} \tag{3.5}$$

$$H(f) = \alpha - e^{-j2\pi(f-f_0)\tau},\ \alpha \le 1,\ \textit{nonminimum-phase} \tag{3.6}$$

So equ. (3.5) is the transfer function at *minimum-phase* and equ. (3.6) is the transfer function at *nonminimum-phase*. The squared response of Rummler channel model at *minimum-phase* and *nonminimum-phase* are

$$|H(f)|^2 = 1 + \beta^2 - 2\beta\cos(2\pi(f-f_0)\tau),\ \beta < 1,\ \textit{minimum-phase} \tag{3.7}$$

$$|H(f)|^2 = 1 + \alpha^2 - 2\alpha\cos(2\pi(f - f_0)\tau),\ \alpha \le 1,\ \textit{nonminimum-phase} \tag{3.8}$$

and the group delay $D(f)$ is given by the derivative of the phase characteristic

$$D(f) = -\frac{1}{2\pi}\cdot\frac{d\phi(f)}{df} = \frac{\tau\beta[\beta - \cos(2\pi(f - f_0)\tau)]}{1 + \beta^2 - 2\beta\cos(2\pi(f - f_0)\tau)},\ \beta < 1,\ \textit{minimum-phase} \tag{3.9}$$

$$D(f) = -\frac{1}{2\pi}\cdot\frac{d\phi(f)}{df} = \frac{\tau[1 - \alpha\cos(2\pi(f - f_0)\tau)]}{1 + \alpha^2 - 2\alpha\cos(2\pi(f - f_0)\tau)},\ \alpha \le 1,\ \textit{nonminimum-phase} \tag{3.10}$$

We have flat fade when

$$|H(f)|^2 = 1 \tag{3.11}$$

That is when $\beta = 0$ or $\alpha = 0$ in equ. (3.7) or (3.8). The Notch Depth *ND* is relative flat fade and is (use equ. (3.7) and (3.8))

$$ND = 10\log\left(\frac{1}{|H(f_0)|^2}\right) = 10\log\left(\frac{1}{\beta^2 - 2\beta + 1}\right),\ \beta < 1,\ \textit{minimum-phase} \tag{3.12}$$

$$ND = 10\log\left(\frac{1}{|H(f_0)|^2}\right) = 10\log\left(\frac{1}{\alpha^2 - 2\alpha + 1}\right),\ \alpha \le 1,\ \textit{nonminimum-phase} \tag{3.13}$$

Solving equ. (3.12) and (3.13) for β and α gives us

$$\beta = 1 - 10^{-\frac{ND}{20}},\ \beta < 1,\ \textit{minimum-phase} \tag{3.14}$$

$$\alpha = 1 - 10^{-\frac{ND}{20}},\ \alpha \le 1,\ \textit{nonminimum-phase} \tag{3.15}$$

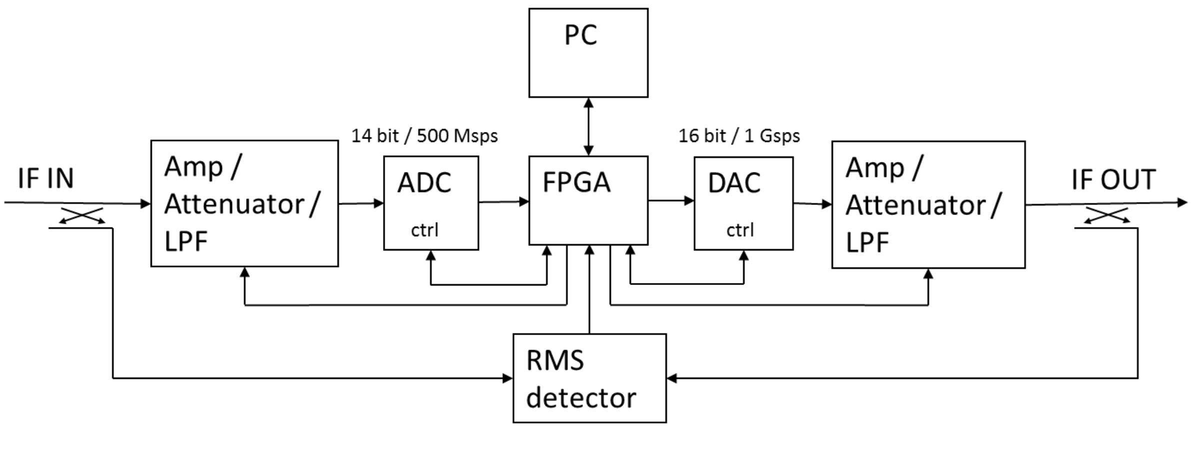

Fig. 3.1 The block schematic of one of the fading emulator modules. The embedded PC controls the GUI of RI 241 and setup the FPGA.

Revision Changes

Rev.	Date	Change	Reference	Approved
1.0	2014-10-14	First revision	Jan Dahl	Tomas Ornstein